Joint Optimization of Pose And Depth Using a Prox-Linear Approach

Master Thesis Presentation

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1. Problem Formulation
2. Optimization
3. Results

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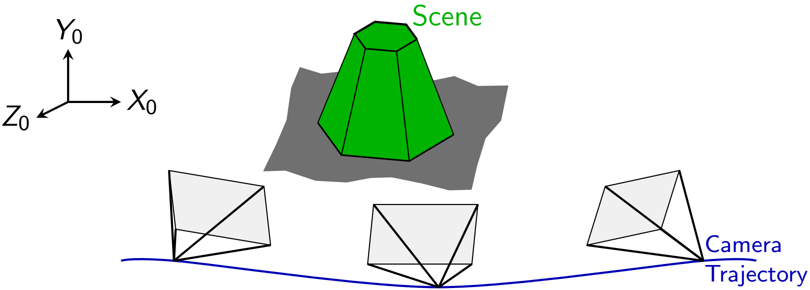
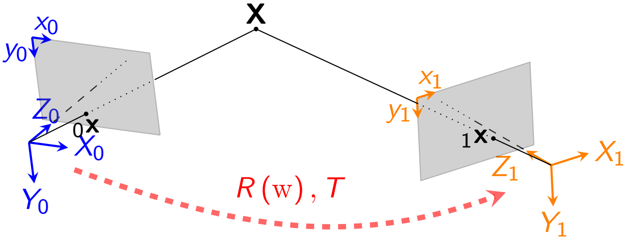
SLAM Overview



Warping Between two Frames

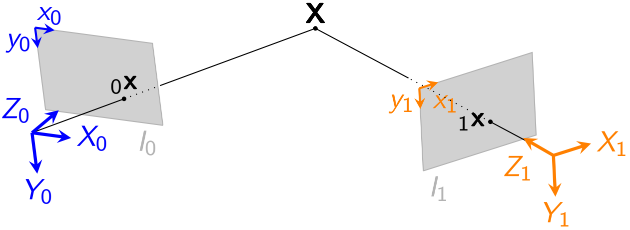
Pinhole Camera: $h\tilde{\mathbf{x}} = K\mathbf{X}$

w: Exponential Coordinates



$$_1\mathbf{x} = \omega_{_0\mathbf{x}}\left(R, T, {_0}\mathbf{h}\right)$$

Photometric Data Term



- Photometric $\rightarrow$ Compare image intensities
- Bilinear/Bicubic interpolation in second image
- Assume Lambertian surfaces
- Dense $\rightarrow$ All valid pixels

Photometric Data Term

Data term for two images

$$E_{data}\left(\mathrm{w}, T, \mathbf{h}\right) = \sum_{\mathbf{x} \in \mathcal{V}_{I_1}} \ell\left(\mathcal{J} I_1\left(\omega_{\mathbf{x}}\left(R\left(\mathrm{w}\right), T, \mathbf{h}\right)\right) - I_0\left(\mathbf{x}\right)\right)$$

- $\mathcal{V}_{I_1}$: Set of valid pixels
- ℓ: Loss function
- $\mathcal{J}$: Interpolation operator
- Extendable for more images

Regularization

Isotropic Huber Regularization with image-driven weights

$$E_{reg}(\mathbf{h}) = \sum_{\mathbf{x} \in \mathcal{P}} \gamma(\mathbf{x}) \left\| D\mathbf{h}(\mathbf{x}) \right\|_h$$

- $\mathcal{P}$: Set of all pixels
- $\gamma(\mathbf{x}) = e^{-\alpha \| D I_0(\mathbf{x}) \|_2^\beta}$
- D: Discrete gradient operator (forward differences)
- Huber norm: $\|x\|_h = \begin{cases} \frac{1}{2h} \|x\|_2^2 & \text{if } \|x\|_2 \leq h \\ \|x\|_2 - \frac{h}{2} & \text{else} \end{cases}$

Joint Optimization Problem

Joint Optimization Problem

$$\min_{\mathbf{w},\mathbf{T},\mathbf{h}} \left\{E_{data}\left(\mathbf{w},\mathbf{T},\mathbf{h}\right) + \lambda_{reg} E_{reg}\left(\mathbf{h}\right)\right\}$$

Advantages

- Only one optimization for pose and depth
- No keypoint selection, all pixels for pose

Difficulties

- Non-linear, Non-Convex
- High-Dimensional

Composite Optimization Problem

Composite Optimization Problem

$$\min_{x} F(x) := g(x) + h(c(x))$$

- $g : \mathbb{R}^d \to \overline{\mathbb{R}}$ and $h : \mathbb{R}^m \to \mathbb{R}$ proper, closed and convex
- $c : \mathbb{R}^d \to \mathbb{R}^m$ be a C^1-smooth

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$$g(\cdot) \quad \leftrightarrow \quad \lambda_{reg} E_{reg}(\mathbf{h}) = \lambda_{reg} \sum_{\mathbf{x} \in \mathcal{P}} \gamma(\mathbf{x}) \left\| D\mathbf{h}(\mathbf{x}) \right\|_h$$

Composite Optimization Problem

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$$h(c(\cdot)) \quad \leftrightarrow \quad E_{data}(\mathrm{w}, T, \mathbf{h}) = \sum_{\mathbf{x} \in \mathcal{V}_{I_1}} \ell\left(\mathcal{J} I_1\left(\omega_{\mathbf{x}}\left(R(\mathrm{w}), T, \mathbf{h}\right)\right) - I_0(\mathbf{x})\right)$$

The Standard Prox-Linear Algorithm

Composite Optimization Problem

$$\min_{x} F(x) := g(x) + h(c(x))$$

Iterative update

$$x^{k+1} = \arg\min_{x \in \mathbb{R}^d} \left\{ g(x) + h\left(c\left(x^k\right) + Jc\left(x^k\right)\left(x - x^k\right)\right) + \frac{1}{2t}\left\|x - x^k\right\|_2^2 \right\}$$

- $Jc\left(x^k\right)$: Jacobian Matrix of c

The Standard Prox-Linear Algorithm

Composite Optimization Problem

$$\min_x F(x) := g(x) + h(c(x))$$

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- $Jc\left(x^k\right)$: Jacobian Matrix of c
- $g \equiv 0,\ h = \frac{1}{2}\sum x_i^2 \quad \Rightarrow$ Levenberg-Marquardt
- $h(x) = x \quad \Rightarrow$ Prox-gradient algorithm

Weighted Proximal Mapping

- $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ closed, proper and convex
- $M \in \mathbb{R}^{n \times n}$ symmetric positive definite matrix ($\to$ diagonal)
- Weighted norm: $\|u\|_M^2 = \langle M^{-1} u, u \rangle$

$$\mathrm{prox}_{Mf}(u) := \underset{v \in \mathbb{R}^n}{\arg\min} \left\{ f(v) + \frac{1}{2} \|v - u\|_M^2 \right\}$$

The Weighted Prox-Linear Algorithm

Composite Optimization Problem

$$\min_{x} F(x) := g(x) + h(c(x))$$

Iterative update

$$x^{k+1} = \underset{x \in \mathbb{R}^d}{\arg\min} \left\{ g(x) + h\left(c\left(x^k\right) + Jc\left(x^k\right)\left(x - x^k\right)\right) + \frac{1}{2}\left\|x - x^k\right\|_{M^k}^2 \right\}$$

The Weighted Prox-Linear Algorithm

Composite Optimization Problem

$$\min_{x} F(x) := g(x) + h(c(x))$$

Iterative update

$$x^{k+1} = \operatorname*{arg\,min}_{x \in \mathbb{R}^d} \left\{ g(x) + h\left(c\left(x^k\right) + Jc\left(x^k\right)\left(x - x^k\right)\right) + \frac{1}{2}\left\|x - x^k\right\|^2_{M^k} \right\}$$

Step widths: $\left(M^k\right)^{-1} = \frac{1}{\zeta^k_{step}} M_0^{-1} + \operatorname{diag}\left({J^k}^T J^k\right)$

Solution of the Sub-Problems

Sub-Problem

$$u^{k+1} = \arg\min_u \left\{ h\left(J^k u - b^k\right) + \lambda_{reg} E_{reg}\left(u\right) + \frac{1}{2}\left\|u - u^k\right\|^2_{M^k} \right\}$$

General Case

- h: Absolute loss, Huber loss
- Standard TV regularization possible
- Solve using preconditioned PDHG

Solution of the Sub-Problems

Sub-Problem

$$u^{k+1} = \arg\min_{u} \left\{ h\left(J^k u - b^k\right) + \lambda_{reg} E_{reg}\left(u\right) + \frac{1}{2} \left\| u - u^k \right\|_{M^k}^2 \right\}$$

Special Case: $h\left(x\right) = \frac{1}{2}\left\|x\right\|^2$

- Linearize Regularization
- $\Rightarrow$ Analytic Solution
- Mixture: Levenberg-Marquardt on data term, gradient descent on regularization

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Experiment 1

Compare joint estimation to pure depth/pose estimation

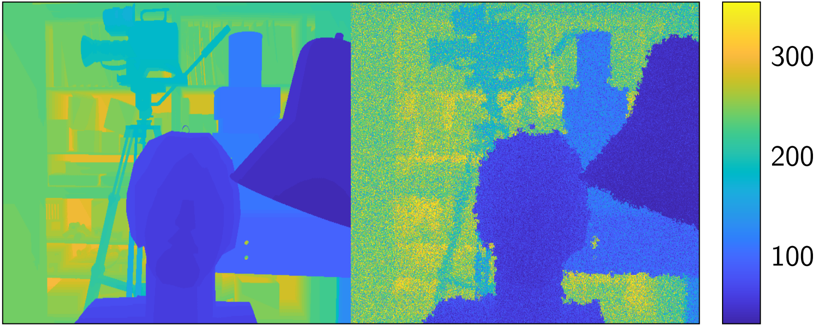
- Absolute data loss
- Pure estimation: Use ground truth
- New Tsukuba data set

Comparison Joint Approach vs. Pure Pose/Depth

Image Pair

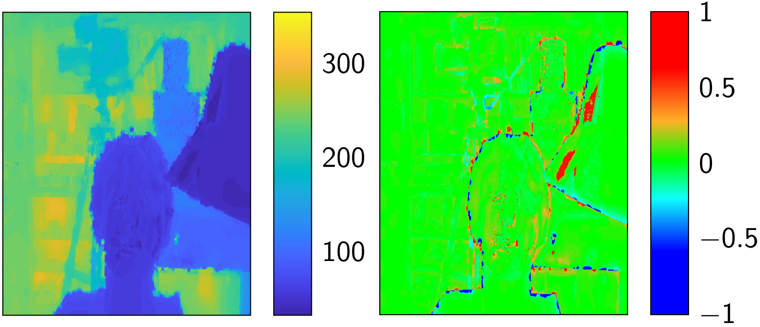
Comparison Joint Approach vs. Pure Pose/Depth

True depth and initial value



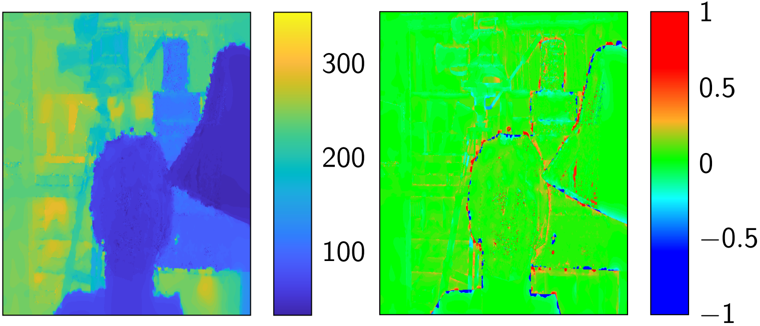
Comparison Joint Approach vs. Pure Pose/Depth

Result Joint Optimization



Comparison Joint Approach vs. Pure Pose/Depth

Result Pure Depth Estimation

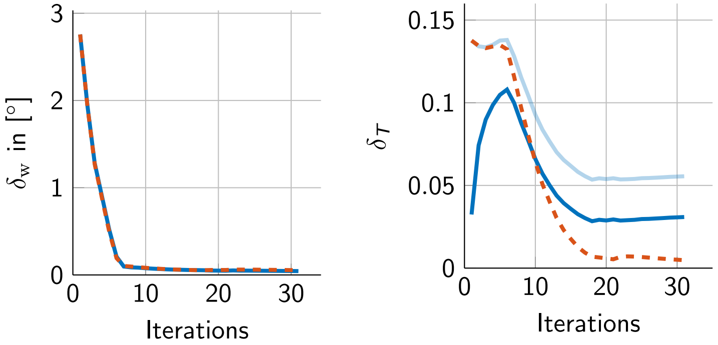


Comparison Joint Approach vs. Pure Pose/Depth

Comparison Results Pose

blue: joint optimization

orange: pure pose optimization



Experiment 2

Compare data loss functions

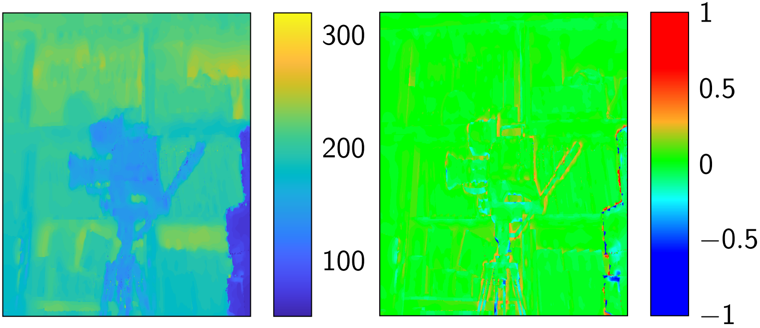
- Absolute loss and special case (quadratic data loss)
- Reduced step widths for special case

Comparison Absolute Loss vs. Special Case

Image Pair

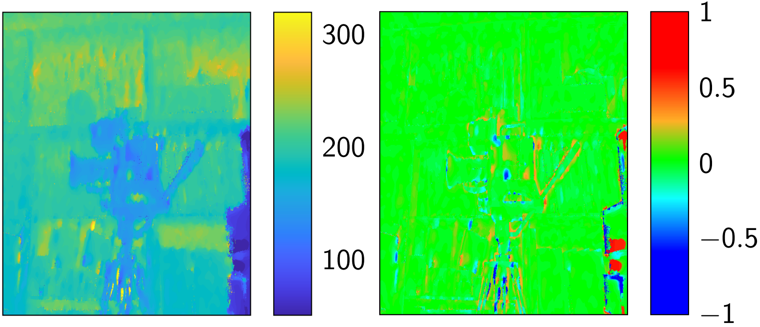
Comparison Absolute Loss vs. Special Case

Result Absolute Loss



Comparison Absolute Loss vs. Special Case

Result Special Case

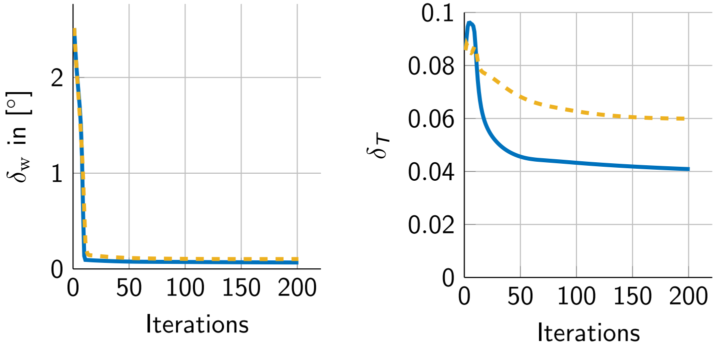


Comparison Absolute Loss vs. Special Case

Comparison Results Pose

blue: absolute loss

yellow: special case (quadratic loss)

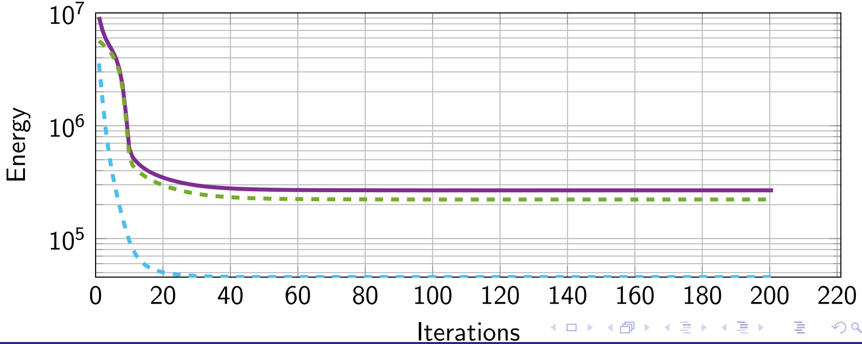


Comparison Absolute Loss vs. Special Case

Energy absolute loss

green: data part

cyan: Regularization part

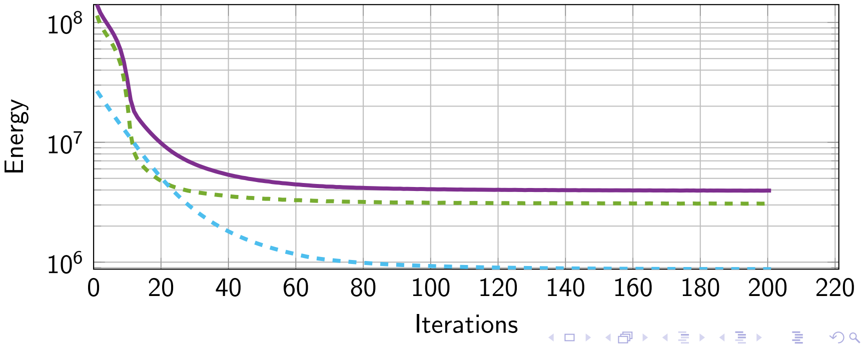


Comparison Absolute Loss vs. Special Case

Energy special case (quadratic loss)

green: data part

cyan: Regularization part



Questions?

Thank You
And
Merry Christmas