Fight ill-posedness with ill-posedness: Single-shot variational depth super-resolution from shading

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Problem Statement

- **Issues with depth maps from commodity RGB-D sensors:**
  noise, quantization effects, missing areas, coarse resolution of depth map

RGB image I Depth image z_0 3D shape

- **Goal:** Given a single (consumer-grade) RGB-D image, leverage the shading information from the color image in order to generate a super-resolution depth image.

Two Ill-posed Problems

Single Depth Super-Resolution (SR)

$$\min_z \|Kz - z_0\|_2^2 + \mathcal{R}(z)$$

- high-resolution depth map z
- low-resolution depth data z_0
- linear downsampling operator K

Shape from Shading (SfS)

$$\min_z \left\| \left\langle l, \begin{bmatrix} \mathbf{n}_z \\ 1 \end{bmatrix} \right\rangle \rho - I \right\|_2^2 + \mathcal{R}(z)$$

$$\mathbf{n}_z = \frac{1}{\sqrt{|f\,\nabla z|^2 + (-z - \langle \mathbf{p}, \nabla z\rangle)^2}} \begin{bmatrix} f\,\nabla z \\ -z - \langle \mathbf{p}, \nabla z \rangle \end{bmatrix}$$

- depth map z
- albedo ρ
- lighting vector l
- RGB image I
- focal length f
- pixel coords. $\mathbf{p}$

Some image   Painter's explanation   Sculptor's explanation   Gaffer's explanation

[Adelson & Pentland; PBI 1996]

References

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Contribution

Fight ill-posedness with ill-posedness to
jointly solve depth super-resolution (SR) and shape-from-shading (SfS)

I z_0 Depth SR   SfS   SR + SfS

Variational Formulation

$$\min_{\rho,l,z} \|\langle l, \mathbf{m}_{z,\nabla z}\rangle \rho - I\|_2^2 + \mu \|Kz - z_0\|_2^2 + \nu \|\mathrm{d}\mathcal{A}_{z,\nabla z}\|_1 + \lambda \|\nabla \rho\|_0$$

Minimal Surface Regularization

$$\|\mathrm{d}\mathcal{A}_{z,\nabla z}\|_1 = \left\| \frac{z}{f^2}\sqrt{|f\,\nabla z|^2 + (-z - \langle \mathbf{p}, \nabla z\rangle)^2} \right\|_1$$

[Graber et al.; CVPR 2015]

Piecewise Constant Albedo Prior

$$\|\nabla \rho\|_0 = \sum_p \begin{cases} 0, & \text{if } |\nabla \rho(p)|_2 = 0, \\ 1, & \text{otherwise.} \end{cases}$$

RGB image   Est. albedo

Numerical Solution

- Introduce auxiliary variable θ:

$$\min_{\rho,l,z,\theta} \|\langle l, \mathbf{m}_\theta\rangle \rho - I\|_2^2 + \mu \|Kz - z_0\|_2^2 + \nu \|\mathrm{d}\mathcal{A}_\theta\|_1 + \lambda \|\nabla \rho\|_0, \quad \text{s.t. } \theta = (z, \nabla z)$$

- Solve linearly constrained problem (nonsmooth & nonconvex) using ADMM:

$$\rho^{(k+1)} = \arg\min_\rho \left\| \langle l^{(k)}, \mathbf{m}_{\theta^{(k)}}\rangle \rho - I \right\|_2^2 + \lambda \|\nabla \rho\|_0,$$

$$l^{(k+1)} = \arg\min_l \left\| \langle l, \mathbf{m}_{\theta^{(k)}}\rangle \rho^{(k+1)} - I \right\|_2^2,$$

$$\theta^{(k+1)} = \arg\min_\theta \left\| \langle l^{(k+1)}, \mathbf{m}_\theta\rangle \rho^{(k+1)} - I \right\|_2^2 + \nu \|\mathrm{d}\mathcal{A}_\theta\|_1 + \frac{\kappa}{2}\left\| \theta - (z, \nabla z)^{(k)} + u^{(k)} \right\|_2^2,$$

$$z^{(k+1)} = \arg\min_z \mu \|Kz - z_0\|_2^2 + \frac{\kappa}{2}\left\| \theta^{(k+1)} - (z, \nabla z) + u^{(k)} \right\|_2^2.$$

$$u^{(k+1)} = u^{(k)} + \theta^{(k+1)} - (z, \nabla z)^{(k+1)}$$

Experimental Results

Quantitative Comparison

[3]   42.8°   42.1°   42.5°

[4]   34.2°   31.5°   31.9°

[2]   39.2°   40.1°   48.0°

Ours   18.9°   16.0°   20.0°

Qualitative Comparison

I

z_0

[3]

[4]

[2]

Ours

Ours (Single View) vs. Multi View State-of-the-Art Methods

I

z_0

z

[5, 1]

Ours

ρ