Variational Uncalibrated Photometric Stereo under General Lighting

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Problem Statement

- **Photometric Stereo** can recover 3D geometry for given images acquired under varying lighting, but remains constrained to amenable modeling and calibration of lighting.

Set of RGB images $\{I^i\}_{i=1}^M$ $(M \geq 3)$ | Depth z | Reflectance ρ | Illumination $\{\mathbf{l}^i\}$

- **Goal:** Uncalibrated photometric stereo under general lighting.

Problem Model

Image Formation Model

The general image formation model:

$$I_c^i(\mathbf{p}) = \int_{\mathbb{S}^2} \rho_c(\mathbf{p}) \ell_c^i(\boldsymbol{\omega}) \max\{\boldsymbol{\omega} \cdot \mathbf{n}(\mathbf{p}), 0\} \, \mathrm{d}\boldsymbol{\omega}, \quad i \in \{1, \dots, M\}, \; c \in \{1, \dots, C\}.$$

Using a second-order spherical harmonic approximation (SHA) on lighting:

$$\int_{\mathbb{S}^2} \ell_c^i(\boldsymbol{\omega}) \max\{\boldsymbol{\omega} \cdot \mathbf{n}(\mathbf{p}), 0\} \, \mathrm{d}\boldsymbol{\omega} \approx \sum_{n=0}^{2} \sum_{m=-n}^{n} (k_n \ell_{n,m}^{i,c}) h_{n,m}(\mathbf{n}(\mathbf{p})),$$

the resulting image formation model appears as:

$$I_c^i(\mathbf{p}) \approx \rho_c(\mathbf{p}) \, \mathbf{l}_c^i \cdot \mathbf{h}[\mathbf{n}](\mathbf{p}),$$
$$\mathbf{h}[\mathbf{n}] = \left[1, \mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_1\mathbf{n}_2, \mathbf{n}_1\mathbf{n}_3, \mathbf{n}_2\mathbf{n}_3, \mathbf{n}_1^2 - \mathbf{n}_2^2, 3\mathbf{n}_3^2 - 1\right]^\top.$$

Normal Parameterization

Perspective projection with intrinsic parameters f_u, f_v, u_0, v_0.
Assuming z is differentiable, the surface normal $\mathbf{n}$ is parameterized by:

$$\mathbf{n}[z](u,v) = \frac{\widetilde{\mathbf{n}}[z](u,v)}{|\widetilde{\mathbf{n}}[z](u,v)|}, \quad \widetilde{\mathbf{n}}[z](u,v) := \begin{bmatrix} f_u \partial_u z(u,v) \\ f_v \partial_v z(u,v) \\ -z(u,v) - (u-u_0)\,\partial_u z(u,v) - (v-v_0)\,\partial_v z(u,v) \end{bmatrix}.$$

References

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[4] Zhipeng Mo, Boxin Shi, Feng Lu, Sai-Kit Yeung, and Yasuyuki Matsushita. Uncalibrated photometric stereo under natural illumination. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2936–2945, 2018.

Contribution

- The **joint estimation** of shape, reflectance and general lighting.
- Recover a **differentiable depth map** using a balloon-like initialization.
- **Robustness** due to Cauchy's M-estimator and Huber-TV regularization.
- Computationally **efficient** because of lagged block coordinate descent.

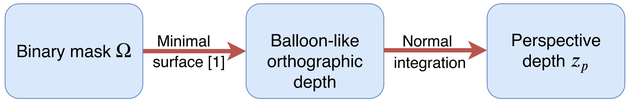
Variational Formulation

$$\min_{\{\rho_c\},\{\mathbf{l}_c^i\},z} \sum_{i=1}^{M} \sum_{c=1}^{C} \int_\Omega \phi_\lambda\Big(\rho_c(u,v)\,\mathbf{l}_c^i \cdot \mathbf{h}[\mathbf{n}[z]](u,v) - I_c^i(u,v)\Big)\,\mathrm{d}u\,\mathrm{d}v + \mu \sum_{c=1}^{C} \int_\Omega |\nabla \rho_c(u,v)|_\gamma \,\mathrm{d}u\,\mathrm{d}v.$$

- *Cauchy's M-estimator*: data fitting which is robust against outliers.
- *Huber Total Variation*: imposes desirable smoothness on albedos.

Solution Pipeline: Depth Initialization → Lagged Block Coordinate Descent

Depth Initialization



Trivial initialization and its result | Balloon-like initialization z_p and its result

Lagged Block Coordinate Descent

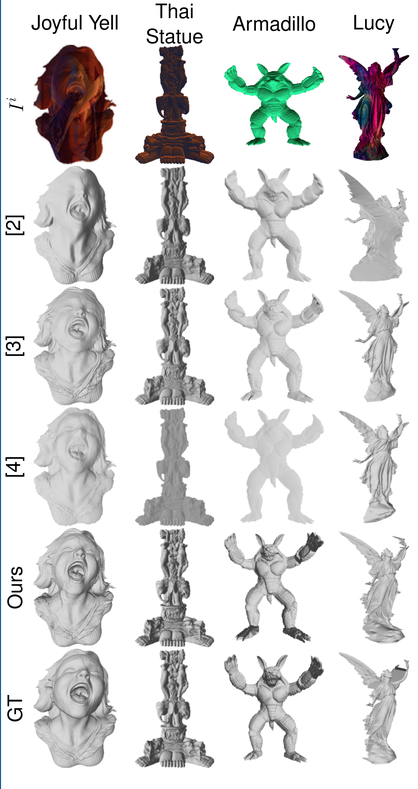
Introduce an auxiliary variable $\theta \in \mathbb{R}^N$ and let $r_{i,c,j}(\theta_j, \rho_{c,j}, \mathbf{l}_c^i, z) := \rho_{c,j}\,\mathbf{l}_c^i \cdot \mathbf{h}_j[\widetilde{\mathbf{n}}_j[z]/\theta_j] - I_{c,j}^i$.

$$\min_{\theta,\{\rho_c\},\{\mathbf{l}_c^i\},z} \sum_{i=1}^{M} \sum_{c=1}^{C} \sum_{j=1}^{N} \phi_\lambda\left(r_{i,c,j}(\theta_j, \rho_{c,j}, \mathbf{l}_c^i, z)\right) + \mu \sum_{c=1}^{C} \sum_{j=1}^{N} |(\nabla \rho_c)_j|_\gamma,$$
$$\text{s.t. } \theta_j = |\widetilde{\mathbf{n}}_j[z]|, \; \forall j \in \{1, \dots, N\}.$$

- Fix $\theta_j = |\widetilde{\mathbf{n}}_j[z]|$ from the previous iteration.
- Update $\{\rho_c\}$: weighted least-squares problem solved by conjugate gradient.
- Update $\{\mathbf{l}_c^i\}$: similar to the $\{\rho_c\}$-update.
- Update z: weighted least-squares problem via linearization of $r_{i,c,j}(\theta_j, \rho_{c,j}, \mathbf{l}_c^i, \cdot)$.

Experimental Results

Synthetic Comparison



	[2]	[3]	[4]	Ours
Median MAE	27.16	21.14	34.06	**9.17**
Mean MAE	34.15	21.18	35.53	**10.72**

Table: Median and mean of the mean angular errors (MAE) over all 36 datasets.

Real-World Comparison