# Efficient Globally Optimal 2D-to-3D Deformable Shape Matching

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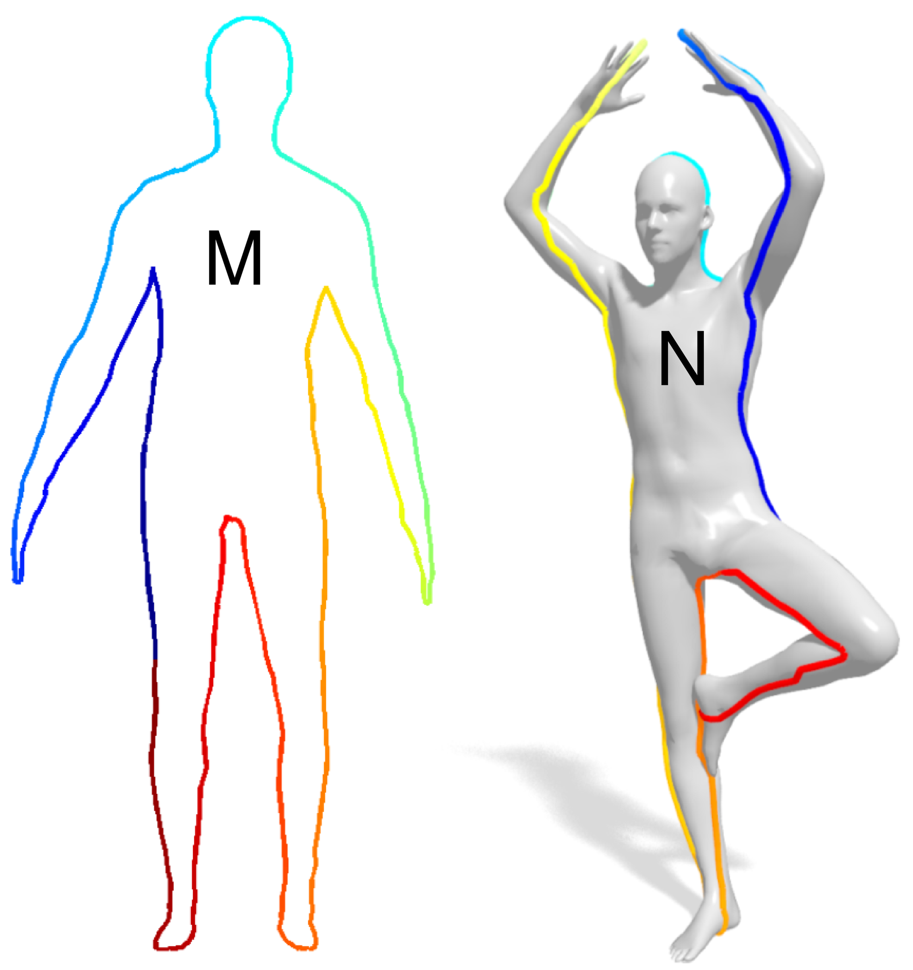
## The Task

Finding a **continuous** matching $\varphi \colon M \rightarrow N$ between a 2D *template shape* $M$ (with $m$ nodes) and a 3D *model shape* $N$ (with $n$ nodes).

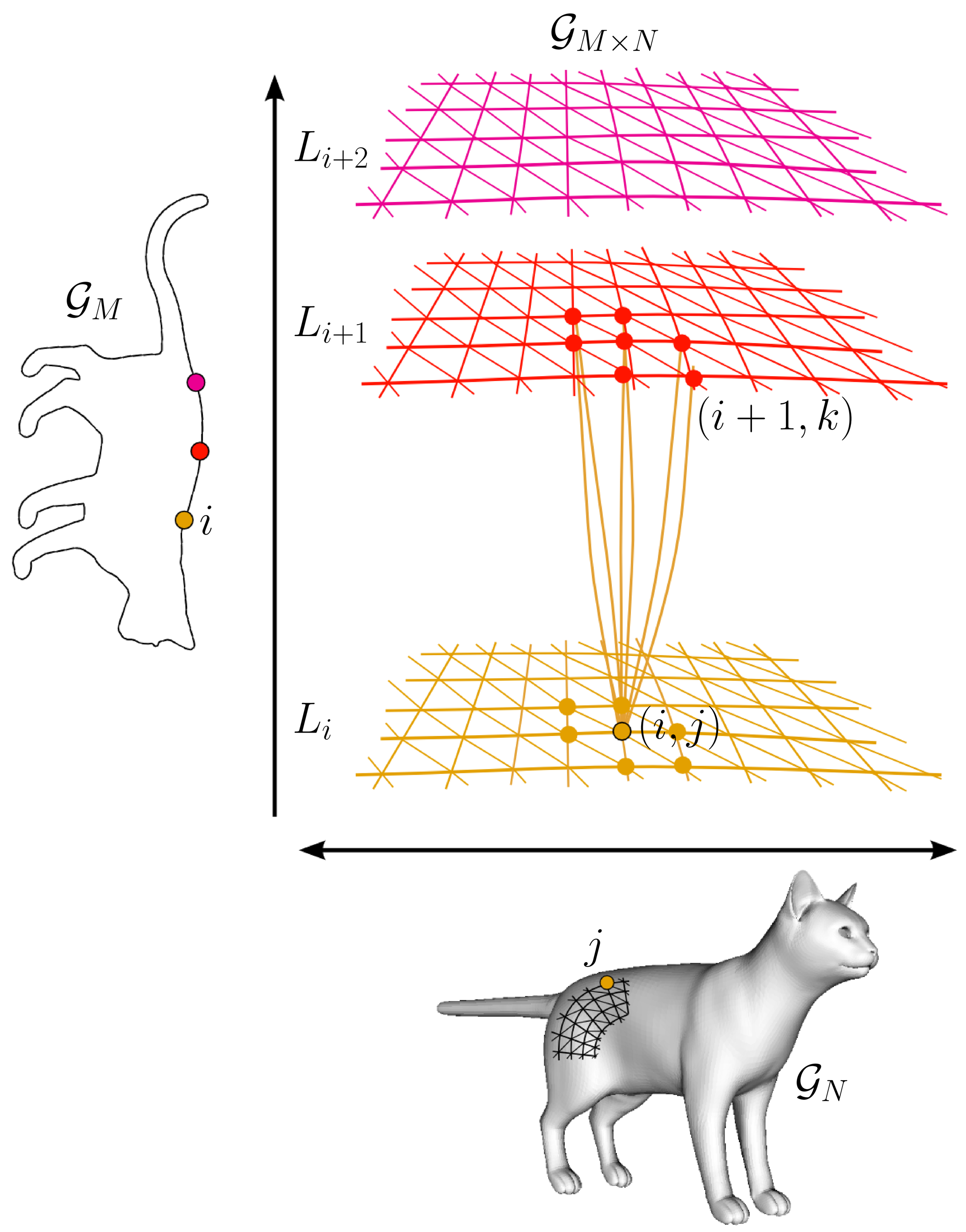



## Contributions

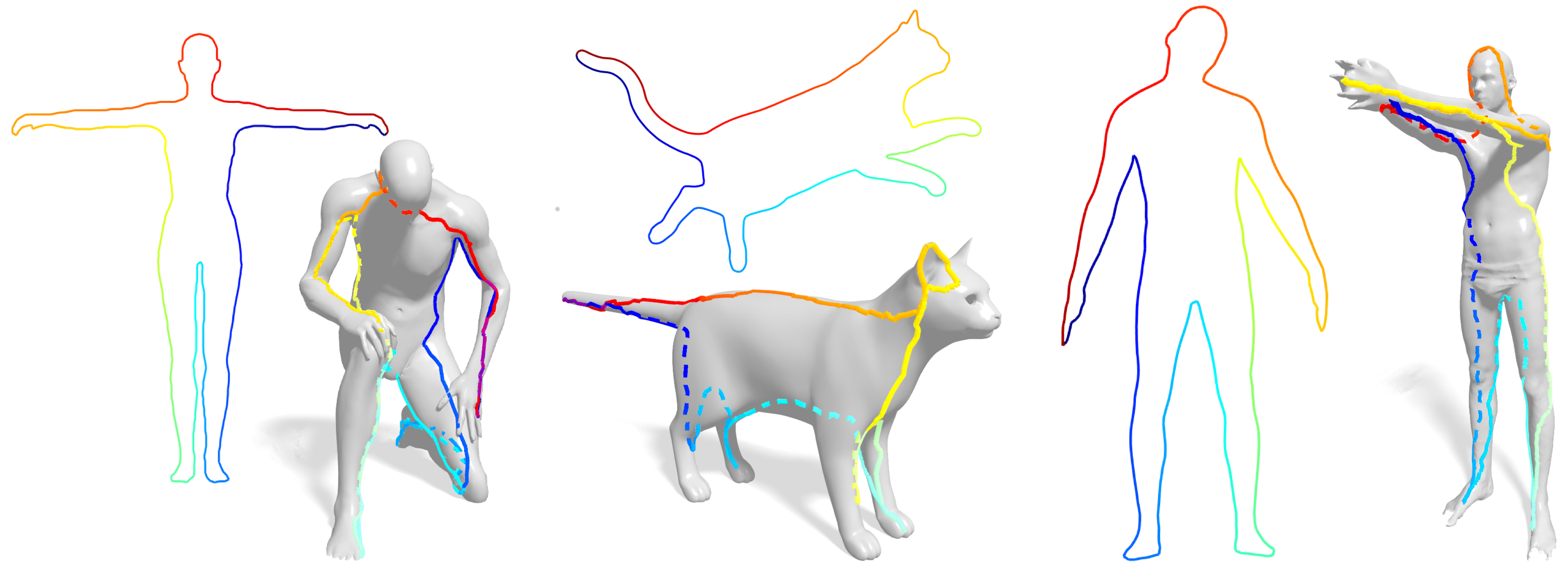
- **Fully automated** Shape Matching method
- **Continuous** matching
- **Globally optimal** solution
- Runtime: $\mathcal{O}(mn^2 \log(n))$
- Comparable 2D and 3D features

## Product Manifold

Each node in the product manifold represents a match between a point on $M$ and $N$. The graph is constructed such that each path through it results in a continuous matching.




## Matchings



## Energy Formulation

Given two comparable features $f_M, f_N$ we minimize the following energy:

$$E(\varphi) = \int_{\Gamma_\varphi} \mathrm{dist}(f_M(s_1), f_N(s_2))\mathrm{d}s \quad (1)$$

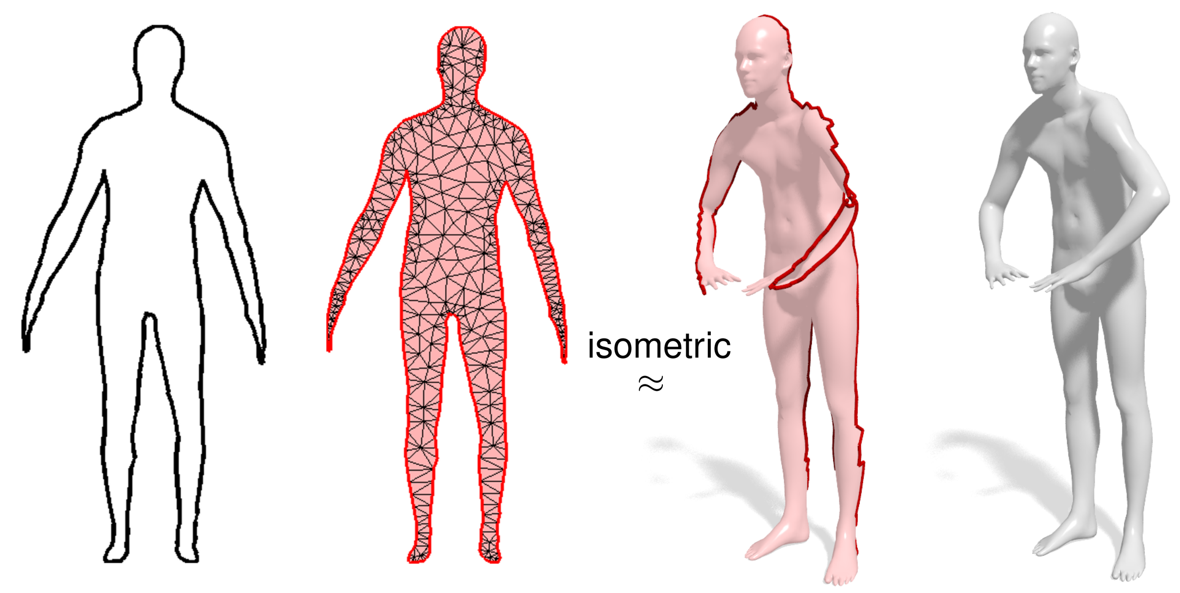
where $\Gamma_\varphi \subset M \times N$ denotes the graph of $\varphi$.

$$\arg\min_{\varphi} E(\varphi) \quad (2)$$

boils down to a **shortest path problem** on the three-dimensional **product manifold** solvable by the Dijkstra algorithm [1,2] and sped up by a branch-and-bound approach [3].

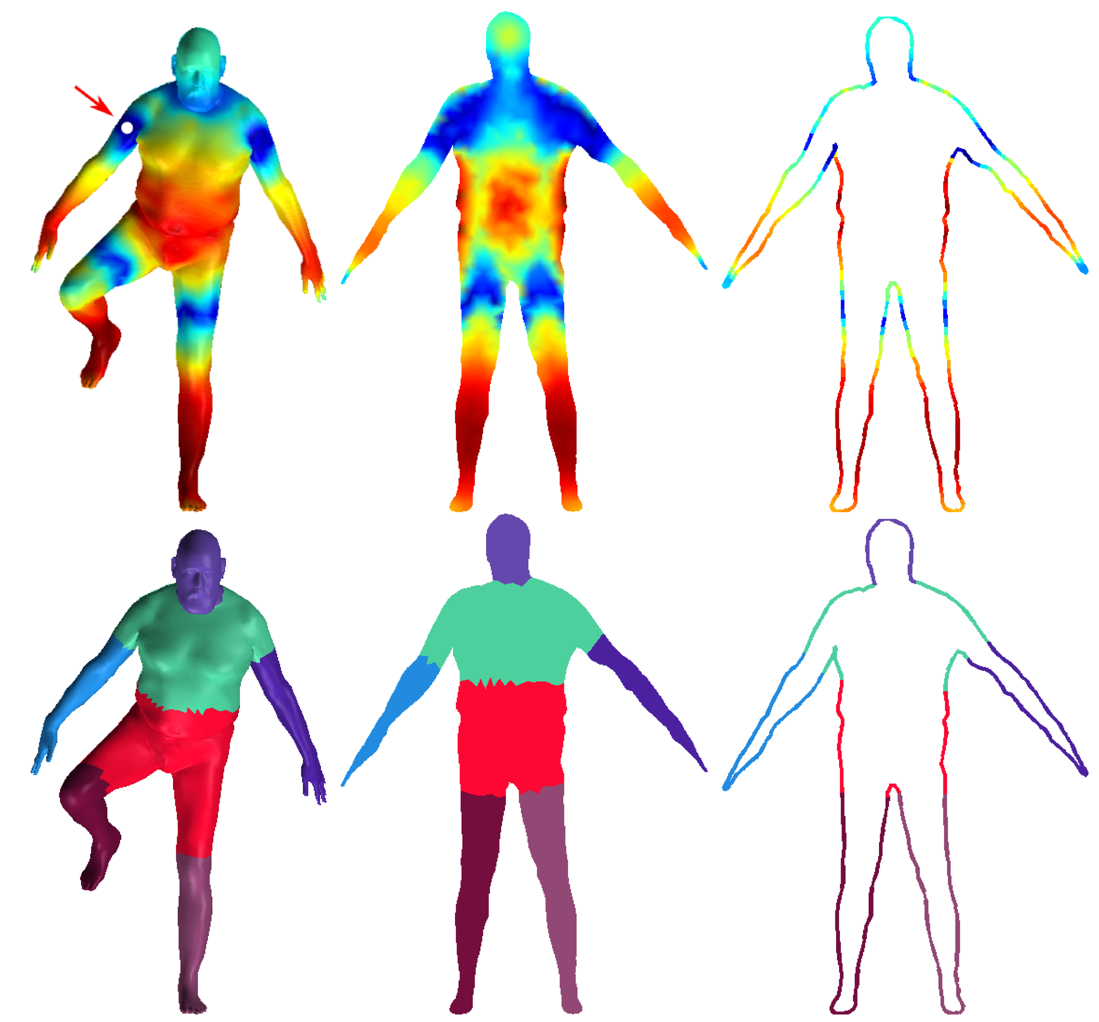
## Spectral Features

We use the scaled HKS, the WKS and Consensus Segmentations as features.




(a) input 2D (b) (c) (d) input 3D

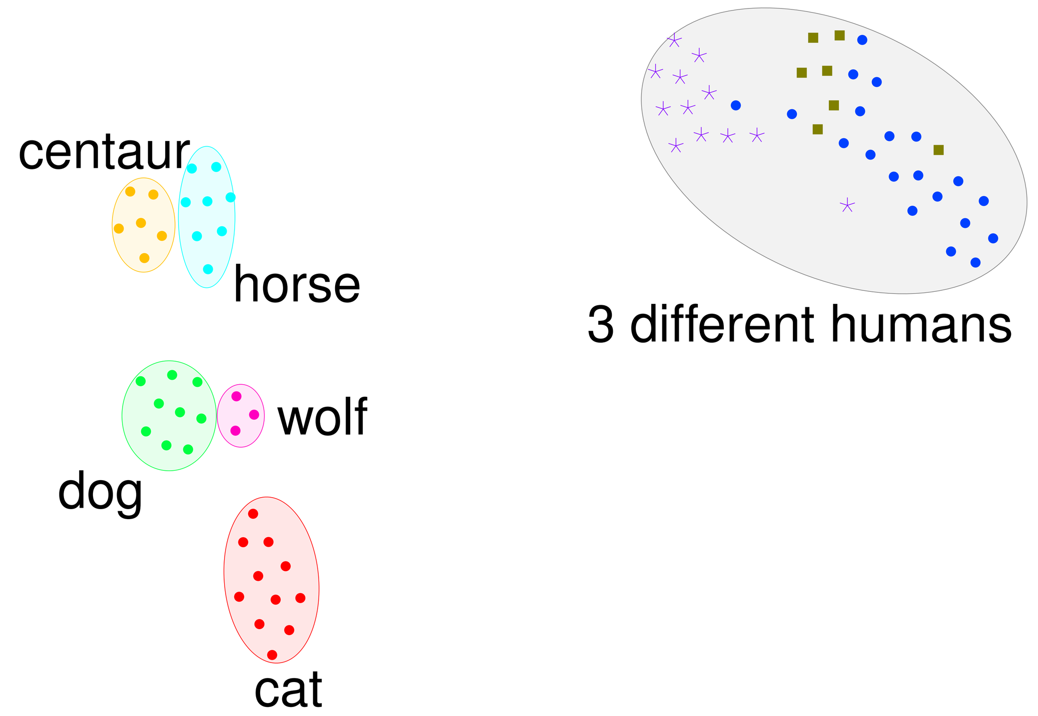
Assumption: Sketches represent all discrimitative extremities of the 3D shape.



## Retrieval Results

The evaluation was done on the whole TOSCA data set and templates of each null pose.

| | **Ours** | ShapeDNA [4] | Consensus [5] |
|---|---|---|---|
| *cat* | **1.0000** | 0.1310 | 0.1050 |
| *human* | **1.0000** | 0.9078 | 0.6532 |
| *dog* | **1.0000** | 0.1066 | 0.3330 |
| *horse* | **0.5062** | 0.0611 | 0.0629 |
| *wolf* | **1.0000** | 0.0379 | 0.0302 |
| MAP | **0.9012** | 0.2489 | 0.2369 |




Embedding produced using all pairwise matching energies by tSNE.

## References

[1] E. W. Dijkstra. A Note on Two Problems in Connexion with Graphs. Numerische Mathematik, 1959.

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[4] M. Reuter, F.-E. Wolter and N. Peinecke. Laplace-Beltrami Spectra as 'Shape-DNA' of Surfaces and Solids. Comput. Aided Design, 2006.

[5] E. Rodolà, S. Rota Bulò and D. Cremers. Robust Region Detection via Consensus Segmentation of Deformable Shapes. Computer Graphics Forum, 2014.

## Contact

More information can be found at vision.in.tum.de/~laehner/Elastic2D3D/ (including C++ Code and the data set)
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