ON THE ANDRÉ MOTIVE OF CERTAIN IRREDUCIBLE SYMPLECTIC VARIETIES

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ABSTRACT. We show that if Y is an algebraic deformation of the Hilbert square of a K3 surface, then the André motive of Y is an object of the category generated be the motive of Y truncated in degree 2.

An irreducible symplectic variety Y is a smooth, projective variety over $\mathbb{C}$ which is simply connected and which admits a nowhere degenerate, holomorphic two-form $\sigma \in H^0(Y, \Omega_Y^2)$. A general principle says that most of the geometry of Y is encoded by the cohomology group $H^2(Y, \mathbb{Z})$ together with the Hodge decomposition and the Beauville–Bogomolov quadratic form. Beauville [B] found two series of examples of irreducible symplectic varieties. Apart from these only two exceptional examples have been discovered by O'Grady [O'G2], [O'G3].

In this note, we study the André motive of irreducible symplectic varieties which are deformation equivalent to the Hilbert scheme of points on a K3 surface or equivalently which are deformations of a smooth, compact moduli space of stable sheaves on a K3 surface. In all even dimensions, there is one family of such deformations, these families build one of Beauville's series of examples. Let Y be such a variety. Denote by $\mathfrak{h}^2(Y)$ the André motive of Y truncated in degree 2. We use results of Markman on the monodromy of moduli spaces of sheaves on K3 surfaces and André's deformation principle to derive

Theorem. *a)The motive of Y is an object of $\langle \mathfrak{h}^2(Y) \rangle$, the category generated by $\mathfrak{h}^2(Y)$.*

b) The motive of Y is an object of the category generated by motives of Abelian varieties.

c) All Hodge classes on Y are motivated, hence absolute in the sense of Deligne.

Item a) can be seen as a motivic manifestation of the above-mentioned principle. Items b) and c) are consequences of a) and of André's results [An2] on degree 2 motives of Hyperkähler varieties.

It has been proved by Arapura [Ar] that the motive of a moduli space Y parametrizing sheaves on a K3 surface S is an object of the category generated by the motive of S. Since the motive of S is also an object of $\langle \mathfrak{h}^2(Y) \rangle$, our result can be seen as a generalization of Arapura's result.

After quickly reviewing André's motives in Section 1, we collect in Section 2 some of Markman's results [M1], [M2] on the cohomology of moduli spaces of sheaves on K3 surfaces and on their monodromy groups. The proof of our result is given in Section 3.

1. André motives

The idea of the category of motives is to provide the target of a universal cohomology functor for smooth, projective varieties. Grothendieck dreamed of a category which should be Tannakian and semisimple. However, Jannsen [J] proved that Grothendieck's category of homological motives can only be semisimple if homological and numerical equivalence of algebraic cycles coincide on all varieties. This is one of Grothendieck's standard conjectures which are widely open.

In order to circumvent the standard conjectures and to obtain nonetheless a Tannakian and semisimple category of motives, André introduced a category in which he formally inverted the dual Lefschetz operator. The basic ingredient in the theory is the notion of a *motivated cohomology class*. Let X be a smooth, projective variety over $\mathbb{C}$. A cohomology class $\alpha \in H^*(X, \mathbb{Q})$ is motivated if there exist a smooth, projective variety Y and algebraic cycles Z_1, Z_2 on $X \times Y$ such that

$$\alpha = p_{1,*}\left([Z_1] \cup \Lambda_{H \times G}[Z_2]\right).$$

Here, $p_1 : X \times Y \to X$ and $p_2 : X \times Y \to Y$ are the projections, and $\Lambda_{H \times G}$ is the dual Lefschetz operator with respect to some product polarization $p_1^*H + p_2^*G$. Clearly, algebraic cohomology classes are motivated. Vice versa, Grothendieck's standard conjectures would imply that motivated cohomology classes are algebraic.

André [An1] proves that the category of motives defined in terms of motivated correspondences is Tannakian and semisimple over $\mathbb{Q}$. The Betti realization which maps a motive to the underlying singular cohomology group is a conservative fibre functor. (Recall that a functor $F : \mathcal{C} \to \mathcal{C}'$ is conservative if a morphism f in $\mathcal{C}$ is an isomorphism if and only if so is $F(f)$.)

One of the big advantages of motivated cohomology classes is that examples are rather easy to produce. This is mainly due to the following result which gives a positive answer to Grothendieck's invariant cycle conjecture in the motivated world.

Theorem 1.1 (André, see [An1], 5.1). *Let $f : \mathcal{X} \to S$ be a smooth projective morphism where S is a smooth, connected, algebraic variety. Let $s \in S$ be a closed point, $m, n \in \mathbb{N}$ and let*

$$\alpha \in H^*(\mathcal{X}_s, \mathbb{Q})^{\otimes n} \otimes \left(H^*(\mathcal{X}_s, \mathbb{Q})^\vee\right)^{\otimes m}$$

be a motivated class which is invariant under a subgroup of finite index of $\pi_1(S, s)$ (acting on $H^*(S, \mathbb{Q})$ via the monodromy representation). Then any translate of α under parallel transport to $H^*(\mathcal{X}_t, \mathbb{Q})^{\otimes n} \otimes (H^*(\mathcal{X}_t, \mathbb{Q})^\vee)^{\otimes m}$ for $t \in S$ is motivated on $\mathcal{X}_t$.

2. Markman's results

Let S be a projective K3 surface, polarized by an ample divisor H. The Todd genus of S is $\mathrm{td}(S) = 1 + 2[x]$ where x is an arbitrary point of S. A square root is given by $\sqrt{\mathrm{td}(S)} = 1 + [x]$.

For a coherent sheaf E on S define the Mukai vector by

$$v(E) = \mathrm{ch}(E)\sqrt{\mathrm{td}(S)} \in H^*(S, \mathbb{Z}).$$

We associate with S a rational weight two Hodge structure

$$\widetilde{H}(S, \mathbb{Q}) := H^*(S, \mathbb{Q}), \quad \widetilde{H}^{2,0}(S) = H^{2,0}(S), \quad \widetilde{H}^{1,1}(S) = H^0(S) \oplus H^{1,1}(S) \oplus H^4(S).$$

There is a natural duality operator

$$D_S : \widetilde{H}(S, \mathbb{Q}) \to \widetilde{H}(S, \mathbb{Q})$$

acting as $(-1)^i\mathrm{id}$ on $H^{2i}(S, \mathbb{Q})$. Since the Künneth components of the diagonal in $S \times S$ are algebraic, D_S is given by an algebraic class.

The Mukai pairing on $\widetilde{H}(S, \mathbb{Q})$ is given by

$$\langle \alpha, \beta \rangle = -\int_S D_S(\alpha) \cup \beta.$$

This is a non-degenerate, symmetric bilinear form of signature $(4+, 20-)$.

Let now $v \in H^*(S, \mathbb{Z})$ be a primitive and effective (cf. [M2, Def. 1.1]) vector. Then by results of Mukai, Huybrechts, O'Grady and Yoshioka, there exist a polarization H on S and a non-empty, smooth, projective variety $X := M_H(v)$ which parametrizes H-stable sheaves with Mukai vector v on S. Moreover, X is an irreducible symplectic variety of dimension $d = \langle v, v \rangle + 2$ which is deformation equivalent to $\mathrm{Hilb}^{\frac{d}{2}}(S)$. We assume that $d > 2$ and for simplicity we assume that X is a fine moduli space.

Let $\mathcal{E}$ be a universal sheaf on $S \times X$. Then $\mathcal{E}$ is uniquely determined up to the twist by the pull-back of a line bundle from X. Denote by $p : S \times X \to S$ and by $q : S \times X \to X$ the projections, let $\pi_{H^2} : H^*(X, \mathbb{Q}) \to H^2(X, \mathbb{Q})$ be the projection in degree 2. Define

$$\varphi'_1 : \widetilde{H}(S, \mathbb{Q}) \to H^2(X, \mathbb{Q}), \quad \alpha \mapsto \pi_{H^2}\big\{q_*\big(\mathrm{ch}(\mathcal{E}) \cup p^*\sqrt{\mathrm{td}(S)} \cup p^* D_S(\alpha)\big)\big\}.$$

According to a result of O'Grady [O'G1], the restriction of φ'_1 to $v^\perp$ is an isomorphism of Hodge structures (even over $\mathbb{Z}$).

We normalize the correspondence $\mathrm{ch}(\mathcal{E})p^*\sqrt{\mathrm{td}(S)}$ following [M2, Lemma 3.1]: let $\eta := \varphi'_1(v)\langle v, v \rangle^{-1} \in H^2(X, \mathbb{Q})$. Put

$$(1) \qquad\qquad u := \mathrm{ch}(\mathcal{E}) \cup p^*\sqrt{\mathrm{td}(S)} \cup q^*\exp(\eta).$$

Then u is independent of the universal sheaf $\mathcal{E}$ and we define

$$\varphi_1 : \widetilde{H}(S, \mathbb{Q}) \to H^2(X, \mathbb{Q}), \quad \alpha \mapsto \pi_{H^2}\big\{q_*(u \cup p^* D_S(\alpha))\big\}.$$

Note that

$$\varphi_1(\alpha) = \varphi'_1(\alpha) - \frac{\langle \alpha, v \rangle}{\langle v, v \rangle}\varphi'_1(v).$$

This implies that $\varphi_1(v) = 0$ and that $\varphi_{1|v^\perp} = \varphi'_{1|v^\perp}$.

Next, we note that φ_1 is an algebraic correspondence. This is, because u and D_S are so and because the projection $H^*(X, \mathbb{Q}) \to H^2(X, \mathbb{Q})$ is algebraic (cf. [Ar] where the conjecture B is shown for X).

Since the standard conjecture B holds for S as well, there is an algebraic right inverse $\psi :$ $H^2(X,\mathbb{Q}) \to \widetilde{H}(S,\mathbb{Q})$ (see [K, Cor. 3.14]). Since the (Mukai-)orthogonal projection $\widetilde{H}(S,\mathbb{Q}) \to \mathbb{Q}v$ is given on $S \times S$ by the class $-(\langle v,v \rangle^{-1} D_S(v)) \otimes v$, the orthogonal projection $\widetilde{H}(S,\mathbb{Q}) \to v^\perp$ is algebraic. Thus we may assume that ψ induces an isomorphism

$$(2) \qquad \psi : H^2(X,\mathbb{Q}) \xrightarrow{\sim} v^\perp \subset \widetilde{H}(S,\mathbb{Q})$$

which is inverse to $\varphi_{1|v^\perp}$.

Let G_v be the fix group of v in $\mathrm{Aut}(\widetilde{H}(S,\mathbb{Z}), \langle\ ,\ \rangle)$. Markman defines two representations of G_v on $H^*(X,\mathbb{Z})$. We will now describe both of them.

1.) Let p_{ij} be the projection from $X \times S \times X$ to the (i,j)-th factor. For $g \in G_v$ set

$$\gamma_g' := (p_{13})_* \Big(p_{12}^* D_{X \times S}\big((\mathrm{id} \otimes g)(^t u)\big) \cup p_{23}^* u \Big)^{-1} \in H^*(X \times X, \mathbb{Q}),$$

where $D_{X \times S}$ is the duality operator acting by $(-1)^i$ on $H^{2i}(X \times S, \mathbb{Q})$ and the class u was introduced in (1). Let $l : H^*(X,\mathbb{Q}) \to H^*(X,\mathbb{Q})$ be the universal polynomial map which takes the Chern character $(r + a_1 + a_2 + \ldots)$ of a coherent sheaf to its total Chern class $(1 + a_1 + (\frac{a_1^2}{2} - a_2) + \ldots)$. Then by definition

$$\gamma_g := \text{degree } d \text{ part of } l(\gamma_g') = \mathrm{c}_d(\gamma_g').$$

Theorem 2.1 (Markman, [M2], Thm. 3.10 and Cor. 3.14). *i) For $g \in G_v$ the correspondence γ_g acts as a (degree-preserving) automorphism on $H^*(X,\mathbb{Q})$.*

ii) The map

$$\gamma : G_v \to \mathrm{Aut}(H^*(X,\mathbb{Q})), \quad g \mapsto \gamma_g$$

is a faithful representation of G_v.

iii) The class $u \in \widetilde{H}(S,\mathbb{Q}) \otimes H^*(X,\mathbb{Q})$ is invariant under the product representation of G_v, where G_v acts on the first factor via the natural representation.

The theorem implies that the algebraic maps

$$(3) \qquad \varphi_i : \widetilde{H}(S,\mathbb{Q}) \to H^{2i}(X,\mathbb{Q}), \quad \alpha \mapsto \pi_{H^{2i}}\big\{ q_*(u \cup p^* D_S(\alpha)) \big\}$$

are G_v-equivariant.

2.) To define the second representation of G_v recall that the space $\widetilde{H}(S,\mathbb{Q})$ has four positive directions. Given any two positive four-spaces F and F', orientations of these spaces can be compared using orthogonal projections. An isometry $g \in \mathrm{Aut}(\widetilde{H}(S,\mathbb{Q}), \langle\ ,\ \rangle)$ is called *orientation preserving* if for an oriented, positive four-space F the space $g(F)$ has the same orientation. This induces the *covariance* or *orientation character*

$$(4) \qquad \mathrm{cov} : G \to \mathbb{Z}/2\mathbb{Z}$$

sending g to 0 or 1 according to whether it preserves orientations or not.

Then Markman defines the representation

$$\gamma_{\mathrm{mon}} : G_v \to \mathrm{Aut}(H^*(X, \mathbb{Q})), \ \ g \mapsto (D_X)^{\mathrm{cov}(g)} \circ \gamma_g$$

where again D_X is the duality operator of X, acting by $(-1)^i \mathrm{id}$ on $H^{2i}(X)$.

The subscript is justified by the following result of Markman. An element $g \in \mathrm{Aut}(H^*(X, \mathbb{Q}))$ is called a *monodromy operator* if there exist a family $\mathcal{X} \to B$ of Hyperkähler manifolds with fibre $\mathcal{X}_b = X$ for some $b \in B$ and a $\widetilde{g} \in \pi_1(B, b)$ such that g is the image of $\widetilde{g}$ under the monodromy representation $\pi_1(B, b) \to \mathrm{Aut}(H^*(X, \mathbb{Q}))$. Let $\mathrm{Mon}(X)$ be the subgroup of $\mathrm{Aut}(H^*(X, \mathbb{Q}))$ generated by monodromy operators.

Theorem 2.2 (Markman, [M2], Thm. 1.6). *The image of the representation $\gamma_{\mathrm{mon}} : G_v \to \mathrm{Aut}(H^*(X, \mathbb{Q}))$ is a normal subgroup of finite index in $\mathrm{Mon}(X)$.*

In particular, since γ and γ_{mon} coincide on the kernel N' of the orientation character, its image $N := \gamma(N')$ in $\mathrm{Aut}(H^*(X, \mathbb{Q}))$ is a subgroup of finite index in $\mathrm{Mon}(X)$.

Following an idea of Beauville, Markman had proved in previous work that the class of the diagonal in $X \times X$ can be expressed in terms of the Chern classes of the universal sheaf $\mathcal{E}$. This implies

Theorem 2.3 (Markman, [M1], Cor.2). *The Künneth factors of the Chern classes of the universal sheaf $\mathcal{E}$ generate $H^*(X, \mathbb{Q})$.*

Summary of results used in the sequel. We have seen that there are homomorphisms $\varphi_i : \widetilde{H}(S, \mathbb{Q}) \to H^{2i}(X, \mathbb{Q})$ and $\psi : H^2(X, \mathbb{Q}) \to v^\perp$ with the following properties:
  i) The φ_i and ψ are induced by algebraic cycles on $S \times X$ resp. on $X \times S$.
  ii) The homomorphism φ_1 induces an isomorphism

$$v^\perp \to H^2(X, \mathbb{Q})$$

whose inverse is ψ.
  iii) There is a subgroup of finite index $N \subset \mathrm{Mon}(X)$ such that the compositions

$$\eta_{i,0} := \varphi_i \circ \psi : H^2(X, \mathbb{Q}) \to H^{2i}(X, \mathbb{Q})$$

are N-equivariant. This follows from Theorem 2.1 and 2.2.
  iv) For $i \geq 2$, the classes $\varphi_i(v) \in H^{2i}(X, \mathbb{Q})$ are N-invariant. Again, this is implied by Theorem 2.1.
  v) The sum of $H^0(X, \mathbb{Q})$, of the image of $\oplus_{i \geq 1} \eta_{i,0}$ and of the $\varphi_i(v)$ generate the cohomology ring $H^*(X, \mathbb{Q})$ as a $\mathbb{Q}$-algebra. This is a consequence of Theorem 2.3.

3. Proof of the Theorem

a) Let $X = M_H(v)$ be as in the previous section, let Y be a fixed algebraic deformation of X. By this we mean that there exists a smooth, projective morphism of connected, smooth, complex algebraic varieties $\mathcal{X} \to B$ which admits X and Y as fibers.

We have to prove that $\mathfrak{h}(Y)$ is an object of $\langle \mathfrak{h}^2(Y) \rangle$. Recall that by definition, this is the smallest full subcategory of the category $\mathcal{M}$ of André motives which contains $\mathfrak{h}^2(Y)$ and the

unit object $\mathbb{1} = \mathfrak{h}(\mathrm{Spec}(\mathbb{C}))$, which is stable under $\otimes$ and under duals and which contains all subobjects resp. quotients in $\mathcal{M}$ of objects in $\langle \mathfrak{h}^2(Y) \rangle$.

The idea is to identify $\widetilde{H}(S, \mathbb{Q})$ with $G(X) = H^2(X, \mathbb{Q}) \oplus \mathbb{Q}$ and to use Markman's results to define a surjection of a sum of products of $G(X)$ to $H^*(X, \mathbb{Q})$ which is monodromy invariant. By André's deformation principle, this will induce a surjection of a motive $\mathfrak{m}(Y)$ to $\mathfrak{h}(Y)$ where $\mathfrak{m}(Y)$ is an object of $\langle \mathfrak{h}^2(Y) \rangle$. Let's make this precise now.

For any fibre V of $\mathcal{X} \to B$, let $\mathfrak{g}_0(V) := \mathfrak{h}^0(V) \simeq \mathbb{1}$ and for $i = 1, \ldots, d = \dim(X)$ define

$$\mathfrak{g}_i(V) := \big(\mathfrak{h}^2(V) \oplus \mathbb{1}(-1)\big)(-i+1).$$

(For $V = X = M_H(v)$, the motive $\mathfrak{g}_i(X)$ plays the role of $\widetilde{\mathfrak{h}}(S)(-i+1) = \big(\mathfrak{h}^0(S)(-1) \oplus \mathfrak{h}^2(S) \oplus \mathfrak{h}^4(S)(1)\big)(-i+1)$.)

Next, we put

$$\mathfrak{m}(V) := \bigoplus_{(i_1,\ldots,i_d) \in \{0,\ldots,d\}^d} \big(\mathfrak{g}_{i_1}(V) \otimes \ldots \otimes \mathfrak{g}_{i_d}(V)\big).$$

Note that $\mathfrak{m}(V)$ is an object of $\langle \mathfrak{h}^2(V) \rangle$ and that $\mathfrak{m}(V)$ can be seen as a submotive of the motive of a variety $Z(V)$ which is a disjoint union of products of V and $\mathbb{P}^1$.

We fix an isomorphism $\eta_0 : \mathbb{1} \to \mathfrak{h}^0(X)$. For $i = 1, \ldots, d$ we will define below morphisms of motives

$$\eta_i : \mathfrak{g}_i(X) \to \mathfrak{h}^{2i}(X)$$

with the following properties:

1) there exists a subgroup N of finite index in $\mathrm{Mon}(X)$ such that η_i is N-invariant.
2) if we define the morphism

$$\eta : \mathfrak{m}(X) \to \mathfrak{h}(X)$$

as the composition of the morphism

$$\bigoplus(\eta_{i_1} \otimes \ldots \otimes \eta_{i_d}) : \mathfrak{m}(V) \to \bigoplus_{(i_1\ldots,i_d) \in \{0,\ldots,d\}^d} \big(\mathfrak{h}^{2i_1}(X) \otimes \ldots \otimes \mathfrak{h}^{2i_d}(X)\big)$$

with the cup-product morphism

$$\bigoplus \big(\mathfrak{h}^{2i_1}(X) \otimes \ldots \otimes \mathfrak{h}^{2i_d}(X)\big) \to \mathfrak{h}(X),$$

then η is surjective.

Assume for one moment, that the η_i are defined. Consider the family $\mathcal{Z} \to B$ which is constructed by letting vary the $Z(V)$ over B. The morphism η corresponds to a motivated cohomology class on $Z(X) \times X$. Property 1) implies that this class is invariant under a subgroup of finite index of the monodromy group of the family $\mathcal{Z} \times_B \mathcal{X} \to B$. By André's Theorem 1.1 we get a surjection $\mathfrak{m}(Y) \to \mathfrak{h}(Y)$. Since $\mathfrak{m}(Y)$ is an object of $\langle \mathfrak{h}^2(Y) \rangle$ and since this category is closed under quotients in $\mathcal{M}$, the proof is reduced to the construction of the η_i.

Let $\eta_{i,0} : \mathfrak{h}^2(X)(-i+1) \to \mathfrak{h}^{2i}(X)$ be the morphism of motives corresponding to the algebraic homomorphism $\eta_{i,0}$ in item iii) in the summary at the end of the last section. Next, we define

$\eta_{i,1} : \mathbb{1}(-i) \to \mathfrak{h}^{2i}(X)$ as the motivated cohomlogy class $\eta_{i,1} = \varphi_i(v) \in H^{2i}(X, \mathbb{Q})$. Finally we define

$$\eta_i := \eta_{i,0} \oplus \eta_{i,1} : \mathfrak{g}_i(X) \to \mathfrak{h}^{2i}(X).$$

Property 1) has been checked in items iii) and iv) at the end of the preceding section.

Property 2) is a direct consequence of item v). There we have seen that the Betti realization of η is surjective. But the Betti realization is a conservative functor. Thus, η is surjective in $\mathcal{M}$. This proves a)

b) is a direct consequence of [An2, Thm. 1.5.1]. This theorem says that the motive $\mathfrak{h}^2(Y)$ of an irreducible symplectic variety Y is an object of $\mathcal{M}(Ab)$, the smallest Tannakian subcategory of the category of André motives which contains the motives of Abelian varieties. The proof of this theorem relies on the Kuga–Satake correspondence. André shows that the Kuga–Satake homomorphism $P^2(Y) \hookrightarrow H^2(A \times A, \mathbb{Q})$ is motivated where $P^2(Y)$ is the primitive part of $H^2(Y, \mathbb{Q})$ with respect to some polarization and A is a Kuga–Satake variety for $P^2(Y)$. Thus, $\mathfrak{p}^2(Y)$, the motive corresponding to $P^2(Y)$, and hence also $\mathfrak{h}^2(Y)$ are objects of $\mathcal{M}(Ab)$.

c) follows from b) and from [An1, Thm. 0.6.2], which says that all Hodge classes on Abelian varieties are motivated. The proof of this theorem uses the deformation principle to reduce first to Abelian varieties with CM, then to Weil classes and finally to products of elliptic curves. $\square$

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